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MIXED-INTEGER CONIC LINEAR PROGRAMMING - CHALLENGES AND PERSPECTIVES

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14. ABSTRACT
Fundamental Disjunctive Conic Cut (DCC) methodology for Mixed-Integer Conic Linear Optimization (MISOCO) was developed. To describe the convex hull of the intersection of a convex set E and a linear disjunction is at the fundamental problem, and that served as the core of solution techniques for Mixed Integer Conic Optimization (MICO). It was proved that if there exists a cone K that has the same intersection with the boundary of the disjunction as the convex set E, then the convex hull is the intersection of E with K. While uniqueness of a DCC is proved for general MICO, the existence of such a cone is difficult to prove for the general case. Thorough analysis of a parametric family of quadrics all to prove the existence and uniqueness of a second order cone, when E is the intersection of an affine space and a second order cone. An efficiently computable method for finding that cone, which provided novel and powerful DCCs for MISOCO that may be used in branch-and-cut algorithms when solving MISOCO problems. All special and degenerate cases are carefully analyzed and easy to compute criteria are developed to compute the DCC for all cases. Limited, but thorough computational experiments gave strong indication of the power of the DCCs.

15. SUBJECT TERMS
Mixed integer conic optimization; Conic optimization; Discrete optimization; Disjunction; Disjunctive conic cut.

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Summary:

Fundamental Disjunctive Conic Cut (DCC) methodology for Mixed-Integer Conic Linear Optimization (MICO) was developed. To describe the convex hull of the intersection of a convex set E and a linear disjunction is the fundamental problem, and that served as the core of solution techniques for MICO. It was proved that if there exists a cone K that has the same intersection with the boundary of the disjunction as the convex set E, then the convex hull of the disjunction is the intersection of E with K. While uniqueness of a DCC is proved for general MICO, the existence of such a cone is difficult to prove for the general case. Thorough analysis of a parametric family of quadrics allows to prove the existence and uniqueness of a second order cone, when E is the intersection of an affine space and a second order cone. An efficiently computable method was developed for finding that cone, which provided novel and powerful DCCs for Mixed Integer Second Order Cone Optimization (MISOCO), which can be used in branch-and-cut algorithms when solving MISOCO problems. All special and degenerate cases are carefully analyzed and easy to compute criteria are developed to compute a DCC for all cases. Limited, but rigorous computational experiments gave strong indication of the power of the DCCs.

Research Results:

In this project the foundations of developing a comprehensive computational theory for efficient solution of MICO and MISOCO problems were established. A comprehensive family of efficiently computable Disjunctive Conic Cuts (DCCs) and Disjunctive Cylindrical Cuts (DCyC) for MISOCO problems. The fundamental idea is that instead of adding linear constraints as customary in Mixed Integer Linear Optimization (MILO), we add Second Order Cones (SOCs) for tightening the relaxed MISOCO problem obtained when one relaxes the integrality constraints.

The first step was to investigate properties and algebraic descriptions of parametric families of quadrics which have fixed intersections with two given hyperplanes. Both the case when the two hyperplanes are parallel and the case when they are nonparallel were studied under the assumption that the feasible set of the SOCO problem is bounded. Later the theory was extended to the case of unbounded intersections as well. Under mild assumptions, we proved that the family of quadrics with the desired properties can be described with only one parameter. Further, we demonstrated that the quadrics are evolving as the parameter changes, and we gave efficiently computable procedures to compute the parameter value which provides the unique second order cone, or cylinder that contains the convex hull of the disjunction.

Then, using the above outlined theory of quadrics, under realistic assumptions, we developed novel, valid disjunctive conic and cylindrical cuts for MISOCO. We showed that under mild conditions one can find a unique second order conic cut K which gives the convex hall of the disjunctive sets. Additionally, we presented a procedure for finding K. By some simple examples we have demonstrated that our DCC is different from Atamturk and Narayan's "nonlinear conic mixed-integer rounding inequality". This comparison demonstrates that our DCC is new, and frequently stronger than the Nonlinear conic mixed-integer rounding inequality.

This way we have made the first complete development of an efficiently computable family of DCC's and DCyC's for MISOCO. The fundamental theory is summarized in paper [1]. The existence of the disjunctive cut for general convex cones, and the efficiently computable construction for MISOCO is presented in paper [2]. All special and degenerate cases are carefully analyzed and easy to compute criteria are developed to compute DCCs and DCyCs for all cases. The complete description of the novel cuts is presented in the technical report [3].

We have made significant progress in building a test-set library for MISOCO problems. The test set library includes randomly generated MISOCO problems, facility location problems from available literature, robust versions of standard MILO test problems, and Round-lot Portfolio Optimization Problems. The novel DCCs for MISOCO may be used in branch-and-cut algorithms when solving MISOCO problems. The experimental software CICLO was developed to perform limited, but rigorous computational experiments. The CICLO solver utilizes continuous SOCO solvers, MOSEK, CPLES or SeDuMi, builds on the open source CHIPS (available in COIN-OR) branch-and-cut software, and implements the addition of a limited number of DCCs and DCyC's within the branch-and-but framework. The results of the experiments gave strong indication of the power of the DCCs. The problem sets and the computational experiments are described in the working papers [4-5]. All the results are summarized in Julio C. Góez's Ph.D. thesis.

Participants:

Tamás Terlaky (PI, Lehigh), Julio Goez (Ph.D. Student, Lehigh), Yu Fu (Ph.D. Student, Lehigh), Sertalp Cay (Ph.D. Student, Lehigh), Imre Pólik (SAS), Ted Ralphs (Lehigh), and Pietro Belotti (Clemson)

Sincerely,

Prof. Tamás Terlaky, Department Chair
George N. and Soteria Kledaras '87 Endowed Chair Professor

Publications

Ph.D. Thesis

J.C. Góez, *Mixed Integer Second Order Cone Optimization: Disjunctive Conic Cuts – Theory and Experiments,* Department of Industrial and Systems Engineering, Lehigh University, Bethlehem, PA, July 2013.
http://phd.ie.lehigh.edu/~jgoez/wp-content/uploads/thesisJCGoez.pdf

Papers:

1. P. Belotti, J.C. Góez, I. Pólik, T.K. Ralphs, and T. Terlaky: On families of quadratic surfaces having fixed intersections with two hyperplanes, *Discrete Applied Mathematics.* http://www.sciencedirect.com/science/article/pii/S0166218X13002461 Appeared Online First, 2013.
2. P. Belotti, J.C. Góez, I. Pólik, T.K. Ralphs, and T. Terlaky: A conic representation of the convex hull of disjunctive sets and conic cuts for integer second order cone optimization. Under revision after first round in *Mathematical Programming,* 2013.

Posters:

1. Belotti, P., Góez, J.C., Pólik, I., Ralphs, T., Terlaky, T.: *Disjunctive Conic Cuts for Second Order Cone Optimization.* INFORMS Annual Meeting, November 5 10, 2010, Austin, Texas, United States.
2. Belotti, P., Góez, J.C., Pólik, I., Ralphs, T., Terlaky, T.: *Mixed Integer Second Order Cone Optimization (MISOCO): Disjunctive Conic Cuts, Theory and Experiments.* Academic Symposium, Lehigh University, Bethlehem, PA, April 4 5, 2013.

Technical Reports, Working Papers:

1. P. Belotti, J.C. Góez, I. Pólik, T.K. Ralphs, and T. Terlaky: On families of quadratic surfaces having fixed intersections with two hyperplanes, Technical Report 11T-007, Dept. Industrial and Systems Engineering, Lehigh University, May 2011.
2. P. Belotti, J.C. Góez, I. Pólik, T.K. Ralphs, and T. Terlaky: A conic representation of the convex hull of disjunctive sets and conic cuts for integer second order cone optimization. Technical Report 12T-009, Dept. Industrial and Systems Engineering, Lehigh University, April 2012. OptimizationOnline:
http://www.optimization-online.org/DB_HTML/2012/06/3494.html
3. P. Belotti, J.C. Góez, I. Pólik, T.K. Ralphs, and T. Terlaky: Disjunctive Conic Cuts for Mixed Integer Second Order Cone Optimization. Technical Report, Dept. Industrial and Systems Engineering, Lehigh University, September 2013. To be completed and submitted October 2013.
4. S.B. Cay, J.C. Góez, and T. Terlaky: Computational experiments with disjunctive Conic Cuts for MISOCO. Technical Report, Dept. Industrial and Systems Engineering, Lehigh University, September 2013. To be completed and submitted Fall 2013.

5. S.B. Cay, J.C. Góez, and T. Terlaky: Generation of Disjunctive Conic and Cylindrical Cuts for Round-lot Portfolio Optimization Problems. Technical Report, Dept. Industrial and Systems Engineering, Lehigh University, September 2013. To be completed and submitted Fall 2013.

Software:

1. *CICLO: Integer conic linear optimization package.* Authors: J.C. Góez, T.K. Ralphs, Y. Fu, and T. Terlaky

Presentations:

1. *Disjunctive Conic Cuts for Second Order Cone Optimization*, MOPTA, August 18 20, 2010, Bethlehem, PA. Presented by J.C. Góez.
2. *Disjunctive conic cuts for second-order cone programming*, STOR Coloquium, November 29, 2010, The University of North Carolina at Chapel Hill, NC. Presented by I. Pólik.
3. *Disjunctive Conic Cuts for Mixed Integer Second Order Cone Optimization (MISOCO).* AFOSR Grantee Conference, Arlington, VA, April 19-20, 2011. Presented by T. Terlaky.
4. *Cone Linear Optimization (CLO): From LO, SOCO and SDO towards mixed integer CLO*. Budapest University of Technology, June 28, 2011, Budapest, Hungary. Presented by T. Terlaky.
5. *Disjunctive Conic Cuts for Mixed Integer Second Order Cone Optimization (MISOCO).* Conference on Simulation and Optimization, June 29 - July 1, 2011, Széchenyi University, Györ, Hungary. Presented by T. Terlaky.
6. *Generating the convex hull of a disjunction for Mixed Integer Second Order Cone Optimization (MISOCO)*, MOPTA 2011, August 17-19, 2011, Bethlehem, PA. Presented by J.C. Góez.
7. *Cone Linear Optimization (CLO): From LO, SOCO and SDO towards mixed integer CLO*. Sino-Japanese Optimization Meeting, Beijing, China, September 26-29, 2011. Presented by T. Terlaky.
8. *Three Decades of Polynomial Time Algorithms for Linear Optimization*. Beijing Jiaotong University, Beijing, China, September 30, 2011. Presented by T. Terlaky.
9. *Cone Linear Optimization (CLO): From LO, SOCO and SDO towards mixed integer CLO*. West Coast Optimization Day, UBC, Kelowna, BC, Canada, October 1, 2011. Presented by T. Terlaky.
10. *Cone Linear Optimization (CLO): From LO, SOCO and SDO towards mixed integer CLO*. ExxonMobil, Clinton, NJ, October 21, 2011. Presented by T. Terlaky.
11. *Cone Linear Optimization (CLO): From LO, SOCO and SDO towards mixed integer CLO*. Department of Mechanical and Industrial Engineering, University of Toronto, ON, Canada, October 28, 2011. Presented by T. Terlaky.
12. *Generating the Convex Hull of Disjunctions in Mixed Integer Second Order Cone Optimization (MISOCO).* INFORMS Annual Meeting, November 13-16, 2011, Charlotte, NC. Presented by J.C. Góez.

13. *Conic representation of the convex hull of disjunctive sets and conic cuts for integer second order cone optimization.* Department of Mathematics and Statistics, University of Calgary, Calgary, AB, Canada, December 13, 2011. Presented by J.C. Góez.
14. *Conic representation of the convex hull of disjunctive convex sets: Application in discrete second-order conic optimization.* Mixed Integer Programming, January 12, 2012, Aussois, France. Presented by P. Belotti.
15. *Generating the Convex Hull of Disjunctions in Mixed Integer Second Order Conic Optimization.* GERAD/Mprime Seminars, February 16, 2012, Montreal, QC, Canada. Presented by J.C. Góez.
16. *Conic Representation of Convex Hull of Disjunctions in Integer Second Order Cone Optimization (ISOCO).* INFORMS Optimization Society Conference, February 24-26, 2012, Coral Gables, FL. Presented by J.C. Góez.
17. *Cone Linear Optimization: From LO, SOCO, SDO Towards Mixed Integer CLO.* Plenary Lecture at the HPSC 2012 Conference, Hanoi, Vietnam, March 4-10, 2012. Presented by T. Terlaky.
18. *Mixed Integer Second Order Conic Optimization (MISOCO): Disjunctive Conic Cuts.* AFOSR Grantee Conference, April 18-20, 2012, Arlington, VA. Presented by T. Terlaky.
19. *Disjunctive Conic Cuts for Second Order Cone Optimization.* IIE Annual Conference and Expo, May 19-23, 2012, Orlando, FL. Presented by T. Terlaky.
20. *Conic representation of the convex hull of disjunctions of convex sets and Conic Cuts for Mixed Integer Second Order Cone Optimization.* International Symposium on Mathematical Programming, August 19-24, 2012, Berlin, Germany. Presented by T. Terlaky.
21. *Computational Issues on Solving Mixed Integer Second Order Cone Optimization Problems.* INFORMS Annual Conference, October 13-17, 2012, Phoenix, AZ, USA. Presented by J.C. Góez.
22. *Disjunctive Conic Cuts.* INFORMS Computing Society Conference, January 7, 2013, Sant Fe, NM. Presented by J.C. Góez.
23. *Disjunctive Conic Cuts for MISOCO* AFOSR Grantee Conference, April 17-19, 2013, Arlington, VA. Presented by T. Terlaky.
24. *Full Characterization of Disjunctive Conic Cuts for MISOCO.* EUROPT Conference, June 26-28, 2013, Florence, Italy. Presented by T. Terlaky.
25. *Full Characterization of Disjunctive Conic Cuts for MISOCO.* EURO-INFORMS Conference, July 1-4, 2013, Rome, Italy. Presented by T. Terlaky.
26. *A Note on Identification of the Optimal Partition of Second Order Cone Optimization Problems.* ICCOPT 2013, July 28 - August 1, 2013, Lisbon, Portugal. Presented by T. Terlaky.
27. *Full characterization of disjunctive-conic-cuts for MISOCO.* MOPTA 2013, August 14 - 16, 2013, Bethlehem, PA, Presented by J.C. Góez.
28. *Generation of Disjunctive Conic and Cylindrical Cuts for Round-lot Portfolio Optimization Problems.* COR@L Seminar, Lehigh University, September 19, 2013, Bethlehem, PA. Presented by S.B. Cay.